

Reliability Analysis of a Three-Unit Gas Lift System: A Comparative Study of Markov and Random Forest Models

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Abstract: Gas lift is extensively used in the oil & gas industry to draw oil/gas from old wells. It comprises a compressor driven by an engine. This paper presents reliability analysis of a system consisting of three gas lifts operating in parallel. Reliability parameters such as availability of the system, busy period of the maintenance team, expected number of maintenance activities, and system profitability are estimated using real industrial maintenance data collected from the three-unit gas lift system. The dataset covers an observation period of four years during which 582 events of failure, inspections, and repairs were documented. The failures are classified into compressor and engine failures for each gas lift, enabling component-level parameter estimation. The data are used to develop a Markov model which is analyzed using regenerative processes. A detailed sensitivity analysis is also carried out to measure the effect of different parameters on the profitability of the system. Finally, a comparative analysis between the results of Markov model and Random Forest model is presented.

Keywords: Gas lift, Markov processes, Reliability, Sensitivity, Random Forest, Regenerative processes.

1 INTRODUCTION

Reliability modeling and analysis have gradually matured into a broad research stream that blends theoretical precision with engineering practice. Modern investigations no longer treat reliability as a fixed attribute; rather, they consider it a dynamic characteristic shaped by probabilistic behaviors, multi-state transitions, and system-level interactions. Tools such as stochastic models, state-space methods, and computational simulations are increasingly used to reflect the nuanced degradation and failure mechanisms of real systems. Beyond quantifying the likelihood of failure, these methods inform maintenance scheduling, optimize resource deployment, and support long-term performance planning. A substantial body of work has emerged in which scholars examine complex industrial systems under varied operating environments, applying diverse assumptions and modeling strategies to capture their reliability more realistically.

B. Parashar and G. Taneja performed a detailed reliability study of a PLC hot standby system [1]. A. G. Mathew et al. analysed the reliability of a two-unit continuous casting plant in parallel combination and different installation capacities [2]. Liang et al. [3] proposed a simplification approach for Markov state-based models to improve the reliability assessment of complex safety systems while reducing computational complexity. Jain et al. [4] presented a reliability and cost optimization framework for fault-tolerant systems with service interruption and reboot mechanisms, demonstrating the impact of fault recovery strategies on overall system performance. S. Song and Y. Deng carried out a reliability study of a three-unit system with parallel combination and one maintenance team [5]. U. Sharma and Drishti presented an analytical study of powder plant system [6]. R. Malhotra et al. presented a novel analysis between two-unit hot and cold standby redundant systems with varied demand [7]. S. Gupta et al. worked on availability and cost analysis of a multistage, multi-evaporator type compressor [8].

H. Kaur and R. Malhotra extensively analysed a two-unit standby autoclave system with inspection and varying demand [9]. J. Friederich and S. Lazarova-Molnar presented reliability analysis of manufacturing systems including overview, challenges and opportunities [10]; whereas Manisha et al. worked on cost benefit analysis of the utensils manufacturing system with preventive maintenance (PM) [11]. Shakuntla et al. [12] presented a reliability analysis of a polytube manufacturing industry using the supplementary variable technique and evaluated important reliability measures under different operating conditions. Kumari et al. [13] investigated the reliability allocation problem using a constraint optimization genetic algorithm, demonstrating the effectiveness of optimization techniques in improving system reliability.

Liang et al. [14] investigated warranty cost analysis for dependent series systems with distinct warranty periods, demonstrating the importance of warranty policies in reliability-based decision-making. Previous work, such as [15], has primarily concentrated on individual components of the gas lift system—most notably, the compressor. While this provides valuable insight into localized reliability characteristics, it does not capture the holistic behavior of the gas lift system, where interdependencies among multiple units play a decisive role in overall performance. In particular, focusing on a single compressor overlooks the redundancy and shared operational dynamics of multiple gas lifts working in parallel. This creates a knowledge gap between component-level reliability assessments and system-level evaluations that are essential for practical decision-making in oil and gas operations. To bridge this gap, the present study investigates the reliability of the complete gas lift configuration consisting of three parallel gas lifts. Moreover, beyond the reliability estimation, this research integrates sensitivity analysis to identify the most influential operational and maintenance parameters on profitability, ensuring that the analysis is not only technically rigorous but also economically meaningful.

By constructing a Markov model grounded in actual industrial maintenance records and analyzing it through regenerative process techniques, the work offers both a realistic and comprehensive framework that advances beyond prior studies. The use of regenerative process theory is justified because the system possesses regenerative points at which the stochastic process probabilistically restarts, thereby satisfying the assumptions required for regenerative analysis. In particular, the study extends existing literature by developing a comprehensive system-level reliability model for multiple gas lifts operating in parallel; integrating industrial data-driven parameter estimation; incorporating profit-based sensitivity analysis to identify key performance drivers; and presenting a comparative perspective between Markov and Random Forest approaches. Random Forest model is employed as a complementary data-driven tool to provide comparative insight into system behaviour, rather than as a primary predictive framework. The model parameters are derived from aggregated industrial maintenance data, the statistical characteristics of which are summarized in Section 2.1.

1.1. Principal Contributions of the Present Study

The contributions of this work extend beyond the development of a reliability model and provide various advances to the field of reliability. First, unlike previous studies that examined the reliability of individual gas-lift components, this work develops a comprehensive analytical reliability model for an integrated three-unit gas lift system operating in parallel, thus enabling system-level assessment of operational performance. Second, the model parameters are estimated from real industrial maintenance data, providing a practical framework connecting theoretical reliability modelling with field data while preserving industrial confidentiality through aggregated statistical reporting. Third, the study derives closed-form expressions for key reliability measures using regenerative processes, allowing engineers to quantify system availability, maintenance workload and frequency, and economic performance within a unified analytical framework. Fourth, the sensitivity analysis identifies the operational parameters strongly influencing system profitability, thereby providing guidance for maintenance prioritization and reliability improvement rather than merely reporting numerical reliability indices. Finally, the study presents a comparative evaluation of analytical Markov modelling and a Random Forest-based data-driven approach using the same industrial dataset. Instead of positioning artificial intelligence as a replacement for reliability modelling, the comparison demonstrates the complementary strengths of stochastic modelling and machine learning for industrial reliability assessment.

2 SYSTEM DESCRIPTION

Following are the system's operating conditions and assumptions:

- The system consists of three gas lifts operating in parallel.
- A gas lift comprises a compressor and an engine.
- Gas lift-1 and gas lift-2 are driven by diesel-based engines, while gas lift-3 is driven by a gas-based engine.
- Only one gas lift fails at a time. The aggregated maintenance records did not indicate significant overlapping failures; therefore, the single-failure assumption adopted in the model is considered reasonable.
- A gas lift may fail due to compressor failure or engine failure.
- Once a gas lift fails, an inspection is carried out.
- Repair activities commence after completion of the inspection process.
- The system returns to its regenerative state following completion of each repair. It is important to note that this regenerative structure relies on the assumption of perfect repair and independence of stochastic processes. In practical systems, these assumptions may be approximations; however, they are widely adopted in reliability modelling to enable tractable analytical solutions.
- Only one repair team is available.
- The maintenance team is assumed to perform only one repair activity at any given time.
- The failure and repair times are assumed to follow exponential distributions for modelling purposes. The exponential distribution assumption is adopted in this study primarily to ensure analytical tractability within the continuous-time Markov and regenerative process framework. The exponential assumption is widely used in reliability engineering for repairable systems where the memoryless property provides a reasonable first-order approximation of failure and repair processes.

Therefore, in this work, the exponential distribution should be interpreted as a modelling assumption supported by standard engineering practice, rather than a statistically validated distribution fit. The state transition matrix describing the permissible transitions among the regenerative states of the three-unit gas lift system is given below. Rows represent the current state, while columns represent the destination state. Non-zero entries denote the corresponding transition rates.

	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9
S_0	0	λ_1	λ_2	λ_3	0	0	0	0	0	0
S_1	0	0	0	0	$p_1\alpha$	$p_2\alpha$	0	0	0	0
S_2	0	0	0	0	0	0	$p_3\alpha$	$p_4\alpha$	0	0
S_3	0	0	0	0	0	0	0	0	$p_5\alpha$	$p_6\alpha$
S_4	γ_1	0	0	0	0	0	0	0	0	0
S_5	γ_2	0	0	0	0	0	0	0	0	0
S_6	γ_3	0	0	0	0	0	0	0	0	0
S_7	γ_4	0	0	0	0	0	0	0	0	0
S_8	γ_5	0	0	0	0	0	0	0	0	0
S_9	γ_6	0	0	0	0	0	0	0	0	0

The description of the states is given in Table 1, while the terms used in the above expression are defined in Table 2.

Table 1. States and their Description

State	Description
S_0	System operational (up state)
S_1	Gas lift-1 failed; under inspection
S_2	Gas lift-2 failed; under inspection
S_3	Gas lift-3 failed; under inspection
S_4	Compressor failure in gas lift-1; under repair
S_5	Engine failure in gas lift-1; under repair
S_6	Compressor failure in gas lift-2; under repair
S_7	Engine failure in gas lift-2; under repair
S_8	Compressor failure in gas lift-3; under repair
S_9	Engine failure in gas lift-3; under repair

For analytical purposes, each state is treated as a regenerative state within the adopted regenerative process framework. Values of various rates for the three-unit gas lift system estimated from the aggregated maintenance data are given in Table 2.

Table 2. Values of Various Rates for the Three-Unit Gas Lift System Estimated from Aggregated Maintenance Data

Rate (/hour)	Value (/hour)
α , inspection rate	0.17846
λ_1 , failure rate for gas lift-1	0.00766
λ_2 , failure rate for gas lift-2	0.00866
λ_3 , failure rate for gas lift-3	0.00966
γ_1 , repair rate of compressor for gas lift-1	0.10305
γ_2 , repair rate of engine for gas lift-1	0.06471
γ_3 , repair rate of compressor gas lift-2	0.11305
γ_4 , repair rate of engine for gas lift-2	0.07471
γ_5 , repair rate of compressor gas lift-3	0.12305
γ_6 , repair rate of engine for gas lift-3	0.08471
p_1 , probability of compressor failure for gas lift-1	0.55
p_2 , probability of engine failure for gas lift-1	0.45
p_3 , probability of compressor failure for gas lift-2	0.60
p_4 , probability of engine failure for gas lift-2	0.40
p_5 , probability of compressor failure for gas lift-3	0.65
p_6 , probability of engine failure for gas lift-3	0.35

2.1. Maintenance Dataset Description and Parameter Estimation

The reliability parameters used in this study are estimated from real industrial maintenance data collected from the three-unit gas lift system. Due to confidentiality constraints, only aggregated statistical information is presented. The dataset covers an observation period of 4 years, during which the system operation and maintenance activities were recorded. Over this period, a total number of 582 events of failure, inspections, and repairs were documented. The failures were classified into compressor and engine failures for each gas lift, enabling component-level parameter estimation. The reliability parameters in Table 1 were estimated using standard statistical methods based on aggregated data, as follows

$$\text{Failure rate of each gas lift unit} = \frac{\text{total number of failures of the unit}}{\text{total operating time}}$$

Similarly, the inspection rate and repair rate were obtained. The probabilities of compressor and engine failures were obtained as the relative frequencies of respective failure types for each gas lift unit. This aggregated approach ensures that the essential statistical characteristics of the maintenance dataset are captured while preserving industrial data confidentiality. The above assumptions and estimated system parameters establish the stochastic framework for the proposed reliability model. Based on this formulation, the transition probabilities and mean sojourn times are derived in the next section.

3 TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

The stochastic behaviour of the three-unit gas lift system is completely characterized by the transition probabilities between regenerative states and the corresponding mean sojourn times. A regenerative state is a state at which the stochastic process probabilistically restarts, making the future evolution independent of the past. Here, $q_{ij}(t)$ denotes the probability density function (pdf) associated with the transition from regenerative state S_i to state S_j , while $Q_{ij}(t)$ denotes the corresponding cumulative distribution function (cdf). Throughout the analysis, ‘*’ represents the Laplace convolution operator and ‘**’ denotes the Laplace–Stieltjes convolution operator. The transition probabilities are defined based on the underlying failure, inspection, and repair rates of the system. From the system description, the following non-zero transition probabilities are obtained.

$$q_{01}(t) = (\lambda_1 e^{-\lambda_1 t})(e^{-\lambda_2 t})(e^{-\lambda_3 t}) = \lambda_1 e^{-(\lambda_1 + \lambda_2 + \lambda_3)t} = \lambda_1 e^{-\lambda t}$$

where $\lambda = \lambda_1 + \lambda_2 + \lambda_3$, $q_{01}(t)$ is the transition of the system from state S_0 to state S_1 . Therefore, the system cannot transition simultaneously to more than one state; the transition probabilities $q_{01}(t)$, $q_{02}(t)$, and $q_{03}(t)$ are derived independently according to the initiating failure event. Accordingly, the transition probability is obtained as shown above. Similarly,

$$\begin{aligned} q_{02}(t) &= \lambda_2 e^{-\lambda t} \\ q_{03}(t) &= \lambda_3 e^{-\lambda t} \\ q_{14}(t) &= p_1 \alpha e^{-\alpha t} \\ q_{15}(t) &= p_2 \alpha e^{-\alpha t} \\ q_{26}(t) &= p_3 \alpha e^{-\alpha t} \\ q_{27}(t) &= p_4 \alpha e^{-\alpha t} \\ q_{38}(t) &= p_5 \alpha e^{-\alpha t} \\ q_{39}(t) &= p_6 \alpha e^{-\alpha t} \\ q_{40}(t) &= \gamma_1 e^{-\gamma_1 t} \\ q_{50}(t) &= \gamma_2 e^{-\gamma_2 t} \\ q_{60}(t) &= \gamma_3 e^{-\gamma_3 t} \\ q_{70}(t) &= \gamma_4 e^{-\gamma_4 t} \\ q_{80}(t) &= \gamma_5 e^{-\gamma_5 t} \\ q_{90}(t) &= \gamma_6 e^{-\gamma_6 t} \end{aligned}$$

Using the definition of nonzero elements p_{ij} [16], the following expressions are obtained.

$$\begin{aligned} p_{01} &= \frac{\lambda_1}{\lambda} \\ p_{02} &= \frac{\lambda_2}{\lambda} \\ p_{03} &= \frac{\lambda_3}{\lambda} \\ p_{14} &= p_1 \\ p_{15} &= p_2 \end{aligned}$$

$$\begin{aligned}
 p_{26} &= p_3 \\
 p_{27} &= p_4 \\
 p_{38} &= p_5 \\
 p_{39} &= p_6 \\
 p_{40} &= p_{50} = p_{60} = p_{70} = p_{80} = p_{90} = 1
 \end{aligned}$$

The following equations are readily verified.

$$p_{14} + p_{15} = p_{26} + p_{27} = p_{38} + p_{39} = 1$$

Using the definition of mean sojourn time μ_i [16], the following expressions are obtained.

$$\begin{aligned}
 \mu_0 &= \frac{1}{\lambda} \\
 \mu_1 &= \mu_2 = \mu_3 = \frac{1}{\alpha} \\
 \mu_4 &= \frac{1}{\gamma_1} \\
 \mu_5 &= \frac{1}{\gamma_2} \\
 \mu_6 &= \frac{1}{\gamma_3} \\
 \mu_7 &= \frac{1}{\gamma_4} \\
 \mu_8 &= \frac{1}{\gamma_5} \\
 \mu_9 &= \frac{1}{\gamma_6}
 \end{aligned}$$

The transition probabilities and mean sojourn times derived above fully define the probabilistic dynamics of the proposed system. These results are now utilized to develop analytical expressions for the principal reliability measures.

4 RELIABILITY ANALYSIS

Having established the stochastic characteristics of the system, this section derives the principal reliability indices using regenerative process theory.

4.1. Availability of the System

Let $A_{si}(t)$ denote the probability that the system is operational at time t when it starts from regenerative state S_i . Let $M_0(t)$ denote the probability that the system remains in the fully operational state S_0 without experiencing any transition up to time t . Based on regenerative process theory and the definition of $A_{si}(t)$ [16], the following expressions are obtained.

$$\begin{aligned}
 A_{s0}(t) &= M_0(t) + q_{01}(t) * A_{s1}(t) + q_{02}(t) * A_{s2}(t) + q_{03}(t) * A_{s3}(t) \\
 A_{s1}(t) &= q_{14}(t) * A_{s4}(t) + q_{15}(t) * A_{s5}(t) \\
 A_{s2}(t) &= q_{26}(t) * A_{s6}(t) + q_{27}(t) * A_{s7}(t) \\
 A_{s3}(t) &= q_{38}(t) * A_{s8}(t) + q_{39}(t) * A_{s9}(t) \\
 A_{s4}(t) &= q_{40}(t) * A_{s0}(t) \\
 A_{s5}(t) &= q_{50}(t) * A_{s0}(t) \\
 A_{s6}(t) &= q_{60}(t) * A_{s0}(t) \\
 A_{s7}(t) &= q_{70}(t) * A_{s0}(t) \\
 A_{s8}(t) &= q_{80}(t) * A_{s0}(t) \\
 A_{s9}(t) &= q_{90}(t) * A_{s0}(t)
 \end{aligned}$$

where $M_0(t) = e^{-\lambda t}$. Taking Laplace Transform of above equations and solving for $A_{s0}^*(s)$, the following expression is obtained.

$$A_{s0}^*(s) = \frac{N_1(s)}{D_1(s)}$$

In steady state, the system availability is given by

$$A_{s0} = \lim_{s \rightarrow 0} s A_{s0}^*(s) = \frac{N_1}{D_1}$$

where

$$N_1 = \mu_0 \quad (1)$$

$$D_1 = \mu_0 + p_{01}(\mu_1 + p_{14}\mu_4 + p_{15}\mu_5) + p_{02}(\mu_2 + p_{26}\mu_6 + p_{27}\mu_7) + p_{03}(\mu_3 + p_{38}\mu_8 + p_{39}\mu_9) \quad (2)$$

4.2. Expected Busy Period of the Maintenance Team

Let $BI_i(t)$ denote the expected cumulative busy period of the maintenance team due to inspection activities when the process starts from regenerative state S_i . Using simple probabilistic arguments and the definition of $BI_i(t)$ [16], the following expressions are obtained.

$$\begin{aligned} BI_0(t) &= q_{01}(t) * BI_1(t) + q_{02}(t) * BI_2(t) + q_{03}(t) * BI_3(t) \\ BI_1(t) &= M_1(t) + q_{14}(t) * BI_4(t) + q_{15}(t) * BI_5(t) \\ BI_2(t) &= M_2(t) + q_{26}(t) * BI_6(t) + q_{27}(t) * BI_7(t) \\ BI_3(t) &= M_3(t) + q_{38}(t) * BI_8(t) + q_{39}(t) * BI_9(t) \\ BI_4(t) &= q_{40}(t) * BI_0(t) \\ BI_5(t) &= q_{50}(t) * BI_0(t) \\ BI_6(t) &= q_{60}(t) * BI_0(t) \\ BI_7(t) &= q_{70}(t) * BI_0(t) \\ BI_8(t) &= q_{80}(t) * BI_0(t) \\ BI_9(t) &= q_{90}(t) * BI_0(t) \end{aligned}$$

where $M_i(t) = e^{-\alpha t}$ where $M_i(t)$ represents the probability that the maintenance team remains continuously occupied with inspection activities while the system stays in inspection state S_i up to time t . Taking Laplace Transform of above equations and solving for $BI_0^*(s)$, the following expression is obtained.

$$BI_0^*(s) = \frac{N_2(s)}{D_1(s)}$$

The expected busy period of the maintenance team (inspection) in steady state, is given by

$$BI_0 = \lim_{s \rightarrow 0} s BI_0^*(s) = \frac{N_2}{D_1}$$

where

$$N_2 = p_{01}\mu_1 + p_{02}\mu_2 + p_{03}\mu_3 \quad (3)$$

D_1 is defined in Equation (2).

Similarly, the following expressions are obtained.

Expected busy period of the maintenance team (Compressor failure in gas lift-1)

$$BC1_0 = \frac{N_3}{D_1}$$

where

$$N_3 = p_{01}p_{14}\mu_4 \quad (4)$$

Expected busy period of the maintenance team (engine failure in gas lift-1)

$$BE1_0 = \frac{N_4}{D_1}$$

where

$$N_4 = p_{01}p_{15}\mu_5 \quad (5)$$

Expected busy period of the maintenance team (compressor failure in gas lift-2)

$$BC2_0 = \frac{N_5}{D_1}$$

where

$$N_5 = p_{02}p_{26}\mu_6 \quad (6)$$

Expected busy period of the maintenance team (engine failure in gas lift-2)

$$BE2_0 = \frac{N_6}{D_1}$$

where

$$N_6 = p_{02}p_{27}\mu_7 \quad (7)$$

Expected busy period of the maintenance team (compressor failure in gas lift-3)

$$BC3_0 = \frac{N_7}{D_1}$$

where

$$N_7 = p_{03}p_{38}\mu_8 \quad (8)$$

Expected busy period of the maintenance team (engine failure in gas lift-3)

$$BE3_0 = \frac{N_8}{D_1}$$

where

$$N_8 = p_{03}p_{39}\mu_9 \quad (9)$$

4.3. Expected Number of Maintenances

Let $RI_i(t)$ denote the expected cumulative number of maintenance actions initiated when the stochastic process starts from regenerative state S_i . From the regenerative state equations and the definition of $RI_i(t)$ [16], the following expressions are obtained.

$$\begin{aligned} RI_0(t) &= Q_{01}(t) ** (RI_1(t) + 1) + Q_{02}(t) ** (RI_2(t) + 1) + Q_{03}(t) ** (RI_3(t) + 1) \\ RI_1(t) &= Q_{14}(t) ** RI_4(t) + Q_{15}(t) ** RI_5(t) \\ RI_2(t) &= Q_{26}(t) ** RI_6(t) + Q_{27}(t) ** RI_7(t) \\ RI_3(t) &= Q_{38}(t) ** RI_8(t) + Q_{39}(t) ** RI_9(t) \\ RI_4(t) &= Q_{40}(t) ** RI_0(t) \\ RI_5(t) &= Q_{50}(t) ** RI_0(t) \\ RI_6(t) &= Q_{60}(t) ** RI_0(t) \\ RI_7(t) &= Q_{70}(t) ** RI_0(t) \\ RI_8(t) &= Q_{80}(t) ** RI_0(t) \\ RI_9(t) &= Q_{90}(t) ** RI_0(t) \end{aligned}$$

Taking Laplace Stieltjes Transform of above equations and solving for $RI_0^*(s)$, the following expression is obtained.

$$RI_0^*(s) = \frac{N_9(s)}{D_1(s)}$$

The expected number of maintenances (inspection) in steady state, is given by

$$RI_0 = \lim_{s \rightarrow 0} s RI_0^*(s) = \frac{N_9}{D_1}$$

where

$$N_9 = p_{01} + p_{02} + p_{03} \quad (10)$$

Similarly, the following expressions are obtained. Expected number of maintenances (compressor failure in gas lift-1)

$$RC1_0 = \frac{N_{10}}{D_1}$$

where

$$N_{10} = p_{01}p_{14} \quad (11)$$

Expected Number of Maintenances (engine failure in gas lift-1)

$$RE1_0 = \frac{N_{11}}{D_1}$$

where

$$N_{11} = p_{01}p_{15} \quad (12)$$

Expected Number of Maintenances (compressor failure in gas lift-2)

$$RC2_0 = \frac{N_{12}}{D_1}$$

where

$$N_{12} = p_{02}p_{26} \quad (13)$$

Expected Number of Maintenances (engine failure in gas lift-2)

$$RE2_0 = \frac{N_{13}}{D_1}$$

where

$$N_{13} = p_{02}p_{27} \quad (14)$$

Expected Number of Maintenances (compressor failure in gas lift-3)

$$RC3_0 = \frac{N_{14}}{D_1}$$

where

$$N_{14} = p_{03}p_{38} \quad (15)$$

Expected Number of Maintenances (engine failure in gas lift-3)

$$RE3_0 = \frac{N_{15}}{D_1}$$

where

$$N_{15} = p_{03}p_{39} \quad (16)$$

4.4. Profit Model

The economic performance of the system is evaluated using a profit function that explicitly separates revenue generation from maintenance-related costs. Thus, the profit generated by the system is given by

$$P = (C_0As_0) - (C_1BI_0 + C_2BC1_0 + C_3BE1_0 + C_4BC2_0 + C_5BE2_0 + C_6BC3_0 + C_7BE3_0) - (C_8RI_0 + C_9RC1_0 + C_{10}RE1_0 + C_{11}RC2_0 + C_{12}RE2_0 + C_{13}RC3_0 + C_{14}RE3_0) \quad (17)$$

where

- C_0 , revenue/time generated by the system = 2500 \$/hour
- C_1 , maintenance team cost/time (inspection) = 15 \$/hour
- C_2 , maintenance team cost/time (compressor failure in gas lift-1) = 25 \$/hour
- C_3 , maintenance team cost/time (engine failure in gas lift-1) = 20 \$/hour
- C_4 , maintenance team cost/time (compressor failure in gas lift-2) = 25 \$/hour
- C_5 , maintenance team cost/time (engine failure in gas lift-2) = 20 \$/hour
- C_6 , maintenance team cost/time (compressor failure in gas lift-3) = 25 \$/hour
- C_7 , maintenance team cost/time (engine failure in gas lift-3) = 20 \$/hour
- C_8 , maintenance cost/time (inspection) = 10 \$/hour
- C_9 , maintenance cost/time (compressor failure in gas lift-1) = 45 \$/hour
- C_{10} , maintenance cost/time (engine failure in gas lift-1) = 40 \$/hour
- C_{11} , maintenance cost/time (compressor failure in gas lift-2) = 45 \$/hour
- C_{12} , maintenance cost/time (engine failure in gas lift-2) = 40 \$/hour
- C_{13} , maintenance cost/time (compressor failure in gas lift-3) = 45 \$/hour
- C_{14} , maintenance cost/time (engine failure in gas lift-3) = 40 \$/hour

The expressions developed in this section provide a complete mathematical framework for evaluating the performance of the three-unit gas lift system. Their practical implications are further explored through sensitivity analysis, which identifies the parameters having the greatest influence on system profitability.

5 SENSITIVITY ANALYSIS FOR PROFIT

While the derived reliability indices quantify the current system performance, they do not reveal the relative importance of individual system parameters. Sensitivity analysis is therefore carried out to identify the parameters that most significantly influence system profitability and to provide guidance for maintenance planning and reliability improvement. Sensitivity analysis quantifies the influence of each model parameter on system performance. Relative sensitivity analysis normalizes the absolute sensitivity with respect to the corresponding parameter value and the system profit, thereby enabling meaningful comparison among parameters having different physical units and magnitudes. Because of the wide range of values for different parameters, relative sensitivity analysis also needs to be conducted for a comparison between the effects of various parameters [17][18]. The results of relative sensitivity analysis for profit generated by the system are presented in Table 3.

Table 3. Relative Sensitivity Analysis for Profit

Parameter (μ)	Sensitivity Analysis $\left(\Delta_{\mu} = \frac{\partial P}{\partial \mu}\right)$	Relative Sensitivity Analysis $\left(z_{\mu} = \frac{\Delta_{\mu} * \mu}{P}\right)$
A	1011.84214	0.10319
λ_1	-22295.41358	-0.09759
λ_2	-20268.06393	-0.10030
λ_3	-18706.83666	-0.10326
γ_1	494.88515	0.02914
γ_2	1023.95858	0.03786
γ_3	507.15062	0.03276
γ_4	771.97692	0.03296
γ_5	517.29252	0.03637
γ_6	586.08379	0.02837

Thus, the descending order in which the parameters influence the profit generated by the system is given in Table 4.

Table 4. Descending Order of Parameter Influence on the Profit

Order	Parameter
1	λ_3
2	α
3	λ_2
4	λ_1
5	γ_2
6	γ_5
7	γ_4
8	γ_3
9	γ_1
10	γ_6

Thus, it can be noted that the failure rate of gas lift-3 has the highest impact on profit generated by the system. Whereas, the profit is least affected by the repair rate of engine for gas lift-3. The sensitivity analysis highlights the parameters that deserve priority from both reliability and maintenance perspectives. Using the parameter values estimated from the industrial dataset, the corresponding numerical reliability indices are presented and interpreted in the following section.

6 RELIABILITY INDICES (MARKOV MODEL)

The expressions obtained in Section 4 are now evaluated using the parameter values estimated from the data. Substituting the values from Table 1 in equations (1)-(17), the following reliability indices are obtained:

Availability of the system = 0.70274

Expected Busy Period of the Maintenance Team (inspection) = 0.10230

Expected Busy period of the maintenance team (compressor failure in gas lift-1) = 0.02873
Expected Busy period of the maintenance team (engine failure in gas lift-1) = 0.03743
Expected Busy period of the maintenance team (compressor failure in gas lift-2) = 0.03230
Expected Busy period of the maintenance team (engine failure in gas lift-2) = 0.03258
Expected Busy period of the maintenance team (compressor failure in gas lift-3) = 0.03586
Expected Busy period of the maintenance team (engine failure in gas lift-3) = 0.02805
Expected Number of Maintenances (inspection) = 0.01826
Expected Number of Maintenances (compressor failure in gas lift-1) = 0.00296
Expected Number of Maintenances (engine failure in gas lift-1) = 0.00242
Expected Number of Maintenances (compressor failure in gas lift-2) = 0.00365
Expected Number of Maintenances (engine failure in gas lift-2) = 0.00243
Expected Number of Maintenances (compressor failure in gas lift-3) = 0.00441
Expected Number of Maintenances (engine failure in gas lift-3) = 0.00238
Profit generated by the system = 1750 \$/hour

The numerical values of reliability indices obtained from the Markov model provide important insights into the operational performance of the gas lift system. Their engineering significance is discussed below. The steady-state availability of the system indicates that the system remains operational approximately 70.27% of the time. From an engineering perspective, this implies that nearly 30% of the total operational time is lost due to failures and maintenance activities, which is relatively high for continuous production systems such as gas lift operations. This level of availability suggests that there is significant scope for performance improvement, particularly through reduction of failure rates or enhancement of maintenance efficiency.

The expected busy period of the maintenance team for inspection indicates that the maintenance personnel are actively engaged in inspection activities approximately 10.23% of the time. This relatively low utilization suggests that inspection activities are not the primary contributor to system downtime. However, it also indicates that the maintenance team has available capacity, which could be utilized for preventive maintenance or condition monitoring to improve system reliability. Similarly, the expected number of maintenance actions related to inspection reflects a relatively low frequency of inspection-triggered maintenance events. This suggests that most failures may be occurring without prior detection during inspection, highlighting a potential limitation in the effectiveness of the inspection process. Enhancing inspection strategies or adopting predictive maintenance techniques could improve early fault detection.

The combined analysis of availability, maintenance frequency, and busy period suggests that system performance is primarily constrained by failure characteristics rather than maintenance capacity. Therefore, reliability improvement efforts should focus more on failure prevention and component reliability enhancement, rather than increasing maintenance resources alone. These engineering insights form the basis for the recommendations presented in the conclusion section. To ensure the credibility of the proposed Markov reliability model, a validation exercise is performed using available aggregated maintenance data and comparative analysis.

Empirical (data-based) availability was estimated directly from the industrial maintenance records. The steady-state availability obtained from the Markov model is compared with the empirical availability estimated from maintenance records. The Markov model predicts an availability of 0.70274, while the observed availability from maintenance data is approximately 0.76538. The reasonably good agreement between these values indicates that the model reasonably captures the system's operational behaviour. The model assumptions and resulting reliability indices are consistent with observed operational trends. In particular:

- Gas lift-3 exhibits higher failure rates and contributes most significantly to downtime
- Inspection and repair activities follow the expected sequence reflected in the model structure

These observations support the structural validity of the Markov representation. However, as an additional validation layer, in the next section the results of the Markov model will be compared with predictions obtained from a Random Forest model trained on the same dataset. It is beneficial to examine the above results from a complementary data-driven perspective. The following section compares the analytical predictions of the proposed Markov model with those obtained using a Random Forest model trained on the same dataset.

7 RELIABILITY INDICES

Although analytical reliability models provide physically interpretable estimates based on stochastic system behaviour, machine learning techniques offer an alternative data-driven approach for predicting system performance. This section presents a comparative analysis between the proposed Markov model and a Random Forest model to examine their consistency and complementary capabilities.

The Random Forest model is used here as a supplementary approach to contrast data-driven predictions with analytically derived results from the Markov model. The developed Random Forest regressor was trained using historical maintenance data collected from the gas lift system. The dataset consists of operational records reflecting failure occurrences, repair activities, and inspection events over the observation period. The input features include estimated failure rates, repair rates, and inspection-related parameters derived from maintenance data. The predicted outputs are selected reliability indices, namely system availability and inspection-related measures. A Random Forest regressor consisting of multiple decision trees was employed to capture non-linear relationships between input parameters and reliability indices.

The Random Forest regressor was implemented using 100 decision trees with other parameters retained at their default values. The model was implemented in a standard computational environment using commonly available machine learning libraries. The Random Forest algorithm inherently reduces overfitting through ensemble averaging across multiple trees. The machine-learning dataset consists of 180 samples constructed from the aggregated maintenance records obtained from the original maintenance events. The dataset is divided into 80% training and 20% testing subsets. A Random Forest regressor is implemented with:

- Number of trees: 100
- Maximum depth: not explicitly restricted
- Number of input features: 11

The model is developed using the Scikit-learn library in Python. The model performance on the test dataset is evaluated using Root Mean Square Error (RMSE), Mean Absolute Error (MAE), and the coefficient of determination (R^2), yielding the following results:

- RMSE = 0.045
- MAE = 0.028
- $R^2 = 0.85$

The predicted reliability indices are presented in Table 5.

Table 5. Random Forest Predictions for Some Reliability Indices

Index	Value
Availability	0.78558
Expected Busy Period of Maintenance Team (inspection)	0.16630
Expected Number of Maintenances (inspection)	0.08291

Index-wise deviation analysis is given in Table 6.

Table 6. Index-wise Deviation Analysis

Reliability Index	Markov	RF	Absolute Error	Relative Error (%)
Availability	0.70274	0.78558	0.08284	11.79%
Busy Period (Inspection)	0.10230	0.16630	0.06400	62.56%
No. of Maintenances (Inspection)	0.01826	0.08291	0.06465	354.21%

The numerical evaluation reveals that the agreement between the two models varies across different reliability indices. The relatively low error in availability ($\approx 11.79\%$) indicates that both models capture the overall system performance consistently. However, significantly higher deviations are observed for inspection-related indices, particularly the expected number of maintenances, where the relative error exceeds 300%. This suggests that the Random Forest model tends to overestimate maintenance-related quantities, likely due to its purely data-driven nature and lack of structural constraints. The aggregate error metrics (MAE and RMSE) indicate moderate overall deviation, while the high Mean Absolute Percentage Error (MAPE) highlights non-uniform error distribution across indices. These results demonstrate that:

- The Markov model provides structurally grounded and conservative estimates
- The Random Forest model captures broader data trends but introduces higher variability

Therefore, the two approaches should be viewed as complementary, with the Markov model serving as a reliable analytical baseline and the Random Forest model offering supportive predictive insights. The error distribution further indicates that discrepancies are concentrated in maintenance-related metrics, rather than system-level availability. The comparative analysis demonstrates the complementary strengths of analytical and data-driven reliability assessment while also highlighting the assumptions underlying each approach.

8 SCOPE AND LIMITATIONS OF THE PRESENT STUDY

The proposed reliability framework is developed under a number of assumptions that enable analytical modelling while remaining representative of actual industrial operating conditions. Failure and repair times are assumed to follow exponential distributions, repairs are considered perfect, and at most one gas lift is assumed to fail at any given time. These assumptions permit the use of continuous-time Markov and regenerative processes, leading to closed-form reliability expressions. Although widely adopted in reliability engineering, they may not fully capture ageing effects, imperfect maintenance, or dependent failure mechanisms encountered in some industrial systems. Furthermore, the model parameters are estimated from aggregated maintenance data because detailed operational data are subject to industrial confidentiality. While the aggregated dataset adequately supports parameter estimation and model validation, access to component-level time histories could facilitate more detailed statistical modelling and enable investigation of alternative lifetime distributions.

The comparison with the Random Forest model is intended to illustrate the complementary roles of analytical and data-driven approaches in reliability assessment rather than to establish predictive superiority of either method. Future work may extend the present framework by incorporating imperfect repair models, non-Markovian degradation processes, alternative lifetime distributions, digital twin technologies, and scenario-based optimization to further improve predictive capability and engineering applicability.

9 CONCLUSIONS

The cost-benefit and sensitivity analyses clearly demonstrate that the system's profitability and reliability are disproportionately influenced by the operational weaknesses of gas lift-3, which is powered by a gas-based engine. In contrast, gas lift-1 and gas lift-2, driven by diesel engines, exhibit greater reliability and lower failure frequency. These findings suggest two promising avenues for improvement. Firstly, replacing the gas-based unit with a diesel-driven alternative may improve system reliability and overall profitability. However, it is acknowledged that the study does not include explicit scenario-based simulation comparisons (e.g., direct replacement cases). Therefore, the recommendation is intended as an indicative engineering suggestion based on observed sensitivity and reliability trends, rather than a definitive prescription. It should be interpreted as a direction for potential improvement, subject to further detailed techno-economic and scenario-based validation in future work. Secondly, the inspection strategy for gas lift-3 should be revised to account for its higher vulnerability to unplanned downtime. One practical alternative is to replace fixed-interval inspection with sensor-based condition monitoring of the health of gas lift-3 to predict impending failures before they occur.

To supplement the analysis, a Random Forest model was utilized to forecast some of the reliability indices based on historical data. The Random Forest predictions demonstrate the potential of machine learning for supporting reliability analysis. Random Forest provides a robust approach for handling non-linear behaviours in failure and repair data, thereby supporting reliability forecasting. The Random Forest model provides useful support for forecasting and trend analysis. The Random Forest model provides complementary insights into system behaviour and highlights the potential of data-driven approaches; however, it is not intended as a replacement for the Markov-based analytical framework.

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ETHICS STATEMENT

This study did not involve human or animal subjects and, therefore, did not require ethical approval.

STATEMENT OF CONFLICT OF INTERESTS

The authors declare no conflicts of interest related to this study.

LICENSING

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